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THEORETICAL CURRENT-VOLTAGE CURVE IN LOW-PRESSURE CESIUM DIODE FOR ELECTRON-RICH EMISSION

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C. M. Goldstein

Lewis Research Center, NASA, Cleveland, Ohio

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While considerable interest has been shown in the space-charge analysis of low-pressure (collisionless case) thermionic diodes,[1-4] there is a conspicuous lack in the presentation of results in a way that allows direct comparison with experiment. The current-voltage curve of this report was, therefore, computed for a typical case within the realm of experimental interest.

The model employed in this computation is shown in Fig. 1 and is defined by the limitating potential distributions (curves a and b). Curve a represents the potential V as a monotonic function of position with a slope of zero at the anode; curve b is similarly monotonic with a slope of zero at the cathode. It is assumed that by a continuous variation of the anode voltage, the potential distributions vary continuously from one limiting form to the other. While solutions for infinitely spaced electrodes[1-3] show that spatially oscillatory potential distributions may exist, they have been neglected in this computation.

McIntyre's formulation of the space charge analysis,[2] specialized to finite electrode separation, has been employed. The following set of equations was solved numerically on an IBM 7090 computer:

$$\xi = 4\left(\frac{\pi}{2kT}\right)^{3/4} m_e^{1/4}(e\ J_{eo})^{1/2}x; \tag{1}$$

$$\eta = \frac{eV}{kT}; \tag{2}$$

$$\alpha \equiv \frac{N_i^+(0)}{N_e^+(0)} = \frac{J_{io}}{J_{eo}}\sqrt{\frac{m_i}{m_e}}; \tag{3}$$

$$F(\eta) = [\eta'(\xi)]^2. \tag{4}$$

The following equations apply to Region A of Fig. 1 for $V_a \leq 0$;

$$F(\eta) = e^{\eta_m} H(\eta - \eta_m) + \alpha\left[G(-\eta) - G(-\eta_m)\right]; \qquad \xi \leq \xi_m \tag{5a}$$

$$= e^{\eta_m} G(\eta - \eta_m) + \alpha\left[G(-\eta) - G(-\eta_m)\right]. \qquad \xi \geq \xi_m \tag{5b}$$

The following equations apply to Region B for $V_a \geq 0$:

$$F(\eta) = e^{\eta_m} H(\eta - \eta_m) - \alpha\left\{e^{-\eta_a}\left[H(\eta_a - \eta_m) - H(\eta_a - \eta)\right]\right.$$

$$\left. + 2\left[G(-\eta_m) - G(-\eta)\right] - 2\left(e^{-\eta_m} - e^{-\eta}\right)\right\}; \qquad \xi \leq \xi_m \tag{6a}$$

$$= e^{\eta_m} G(\eta - \eta_m) - \alpha\left\{e^{-\eta_a}\left[H(\eta_a - \eta_m) - H(\eta_a - \eta)\right]\right.$$

$$\left. + 2\left[G(-\eta_m) - G(-\eta)\right] - 2\left(e^{-\eta_m} - e^{-\eta}\right)\right\}; \qquad \xi \geq \xi_m,\ \eta \leq 0 \tag{6b}$$

$$= e^{\eta_m} G(\eta - \eta_m) - \alpha\left\{e^{-\eta_a}\left[H(\eta_a - \eta_m) - H(\eta_a - \eta)\right]\right.$$

$$\left. + 2G(-\eta_m) - 2e^{-\eta_m} + 2\right\}. \qquad \xi \geq \xi_m,\ \eta \geq 0 \tag{6c}$$

Thus,

$$G(\eta) = e^{\eta}\left[1 - E(\eta)\right] + 2\sqrt{\frac{\eta}{\pi}} - 1; \tag{7}$$

$$H(\eta) = e^{\eta}\left[1 + E(\eta)\right] - 2\sqrt{\frac{\eta}{\pi}} - 1; \tag{8}$$

$$E(\eta) = \mathrm{erf}\sqrt{\eta} = \frac{2}{\sqrt{\pi}}\int_0^{\sqrt{\eta}} e^{-t^2}\, dt, \tag{9}$$

where T is the cathode temperature, k is Boltzmann's constant, m_e and m_i represent electron and ion masses, respectively, e is charge on electron, J_{eo} and J_{io} are electron and ion emission currents, respectively, V is potential measured with respect to the cathode, $N_e^+(0)$ and $N_i^+(0)$ are the electron and ion emission densities, respectively, and subscript m refers to potential minimum.

The three parameters ξ_s, η_a, and α are determined by experimental conditions: ξ_s is the nondimensional spatial coordinate (Eq. (1)) evaluated at the separation distance s and η_a is the nondimensional potential (Eq. (2)) evaluated at the anode potential. Note that for given separation distance s, ξ_s^2 is proportional to the electron saturation current J_{eo}.

The computed current-voltage curve for a cesium diode $\xi_s = 10$ and $\alpha = 0.2$ is presented in Fig. 2. Since other authors use different parameters in place of ξ_s for analyzing experimental results, a comparison of three different forms is presented in Table I.

TABLE I

COMPARATIVE VALUES OF EXPERIMENTAL PARAMETER ξ_s

Symbol	Reference	Comparative value
ξ_s	2	10.0
u_0^2	4	11.9
R	5	100

In Fig. 2(a) the curve has been plotted in the convenient form described by Houston and Webster.[5] The Maxwell-Boltzmann line and the vacuum current-voltage curve ($\xi_s = 10$, $\alpha = 0$) have also been plotted for comparison. The same computed current-voltage curve has been plotted in a form suggested by Nottingham,[4] along with his "master" and "universal-limiting" curves, in Fig. 2(b).

Of particular interest is the change in slope of the current-voltage curve in the region of zero anode potential ($\eta_a = 0$). A qualitative understanding of this phenomenon can be realized by reference to Fig. 1. Ions will first be reflected back to the cathode when the anode potential becomes positive ($\eta_a > 0$); hence, the increased slope of the current-voltage curve is caused by the enhanced removal of the space-charge barrier by the reflected ions.

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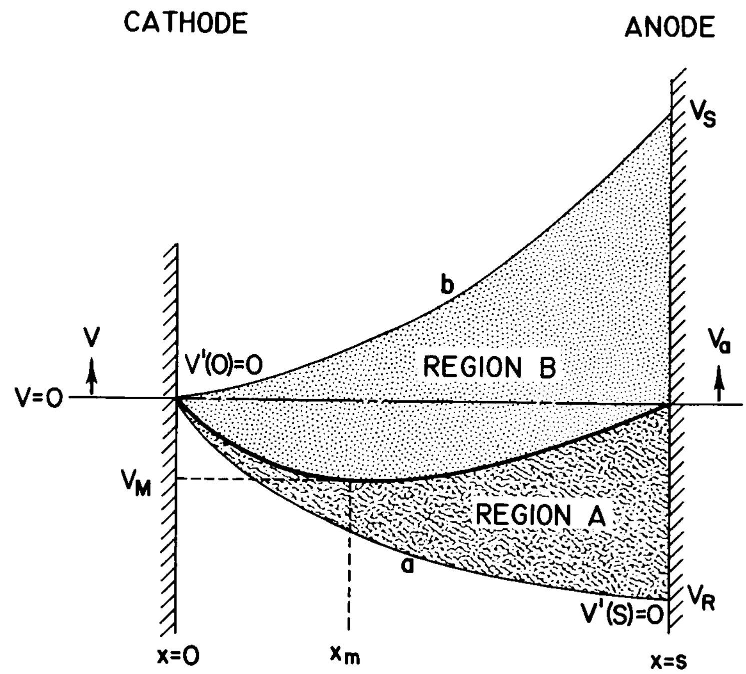
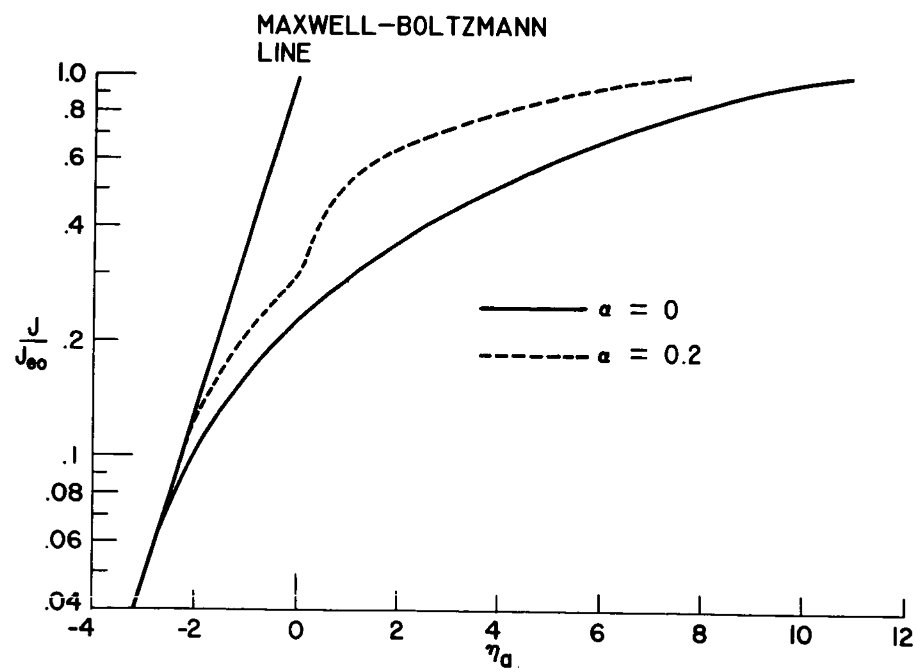
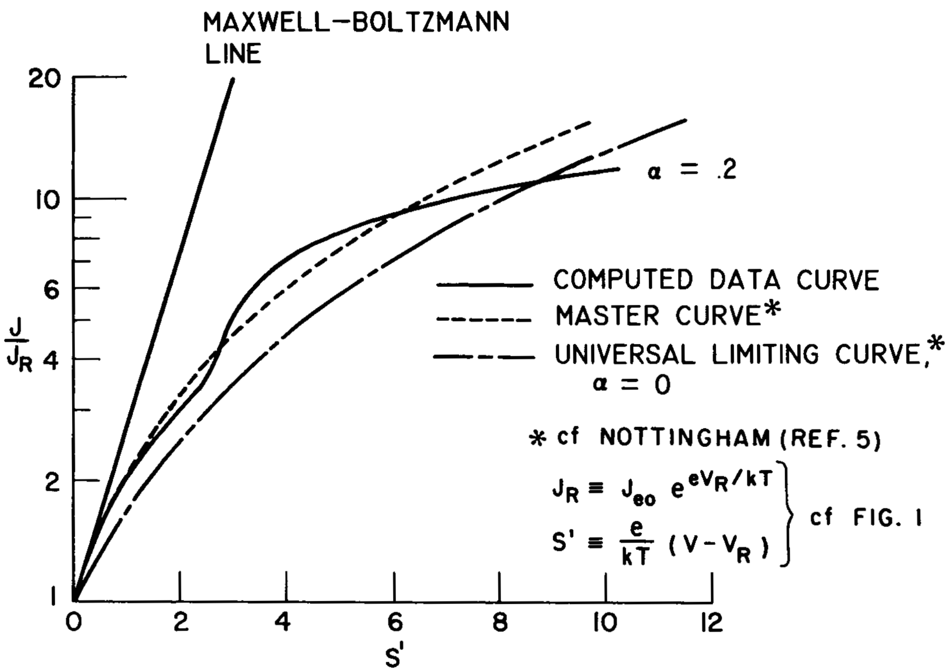


Fig. 1. - Potential model.



(a) Normalized w.r.t. J_{eo}.

Fig. 2. - Computed current-voltage curve. $\xi_s = 10$.



(b) Normalized w.r.t. J_R.

Figure 2. - Concluded. Computed current-voltage curve. $\xi_s = 10$.